FYI Using Hubble's Law to Determine the Age of the Universe

Hubble's Law is not only extremely useful in measuring the distances to the most remote observable galaxies, but also for estimating the age of the universe.

The galaxies that are near the edge of the observable universe have been expanding away from us, and we away from them, for the lifetime of the universe. To simplify the calculations, we will assume that the speed of expansion is constant; then we can say the distance that has grown between Earth and a given distant galaxy is equal to the speed of expansion multiplied by the time the universe has existed. Of course the galaxy and Earth did not exist in the beginning of time, but the space in which they each lie has been expanding since then, so the distance that has grown between them has been increasing. To estimate how long the universe has been expanding (the age of the universe), follow these steps:

$$v = d / t$$

$$H_0 \times d = d / t$$

$$H_0 \times d \times t = d$$

$$t = \frac{d}{H_0 \times d}$$

$$t = \frac{1}{H_0}$$

Step By Step

For simplicity, assume a constant expansion velocity, which can be written as $v = d/t$.

Replace the v with a value from a simple rearrangement of Hubble's Law: $v = H_0 \times d$.

Rearrange the terms to solve for t.

When performing calculations such as those above, it is important to make sure the units of your final quantity, in this case t, make sense. The units of H_0 used in this book are km/s per million light-years. In order to use this value to calculate the age of the universe, we must make all distance units the same throughout the equation so that they will cancel out.

Remember, 1 million light-years = 9.5×10^{18} km. To convert the units of H_0 from km/s/million light-years to km/s/km, divide the original value of H_0 by 9.5×10^{18}.

Example: Find the age of the universe for a Hubble constant of H_o = 23 km/s per million light-years.

$$H_o = \frac{23 \text{ km/s}}{1 \text{ million ly}} \times \frac{1 \text{ million ly}}{9.5 \times 10^{18} \text{ km}} = \frac{23 \text{ km/s}}{9.5 \times 10^{18} \text{ km}}$$

$$= 2.4 \times 10^{-18} \frac{\text{km/s}}{\text{km}} = 2.4 \times 10^{-18} \text{ s}^{-1}$$

Step By Step

Convert the units of H_o from light-years to kilometers, canceling the units and doing the division. (There are 9.5 x 10^{18} kilometers in one million light-years.)

$$t = \frac{1}{H_o} = \frac{1}{2.4 \times 10^{-18} \text{ s}^{-1}} = 4.2 \times 10^{17} \text{ s}$$

Solve for t, using the value you just calculated for H_o.

$$t = 4.2 \times 10^{17} \text{ s} \times \frac{1 \text{ year}}{3.15 \times 10^{7} \text{ s}} = 1.3 \times 10^{10} \text{ years}$$

Convert the units of t from seconds to years. (There are 3.15 x 10^7 seconds in a year.)

So, an H_o of 23 km/s per million light-years yields an age of approximately 13 billion years for the universe.

Checking In

1. Explain what the units of H_o represent and how they indicate that 1/ H_o = the age of the universe.
2. Suppose measurements showed H_o = 35 km/s/million light-years. What would the predicted age of the universe be?

Image of Hoag's Object – a ring galaxy